8.15 INTERFERENCE IN THIN FILMS

Newton and Hooke observed and developed the interference phenomenon due to multiple reflections from the surface of thin transparent materials. Everyone is familiar with the beautiful colours produced by a thin film of oil on the surface of water and also by the thin film of a soap bubble. Hooke observed such colours in thin films of mica and similar thin transparent plates. Newton was able to show the interference rings when a convex lens was placed on a plane glass-plate. Young was able to explain the phenomenon on the basis of interference *between light reflected from the top and the bottom surface of a thin film.* It has been observed that interference in the case of thin films takes place due to (1) reflected light and (2) transmitted light.

3.9 Interference in thin film of uniform thickness

The beautiful colours produced by a thin film of oil on the surface of water and also by the thin film of soap bubble, can be explained by interference phenomenon in thin films.

Let us consider a thin film of refractive index μ and uniform thickness t (Fig. 3.13). Further, let a ray AB of monochromatic light of wavelength λ be incident on this film with the angle of incidence i. This ray is partly reflected along BC and partly refracted along BD at an angle r. The ray BD is again partly reflected from the lower surface of the film along DE and partly refracted or transmitted along DM. The reflected part DE is again partly reflected and partly refracted at point E along EG and EF directions. Similarly, the reflected part EG will be again reflected and refracted from the lower surface of thin film along GL and GN direction. This process will be continue.

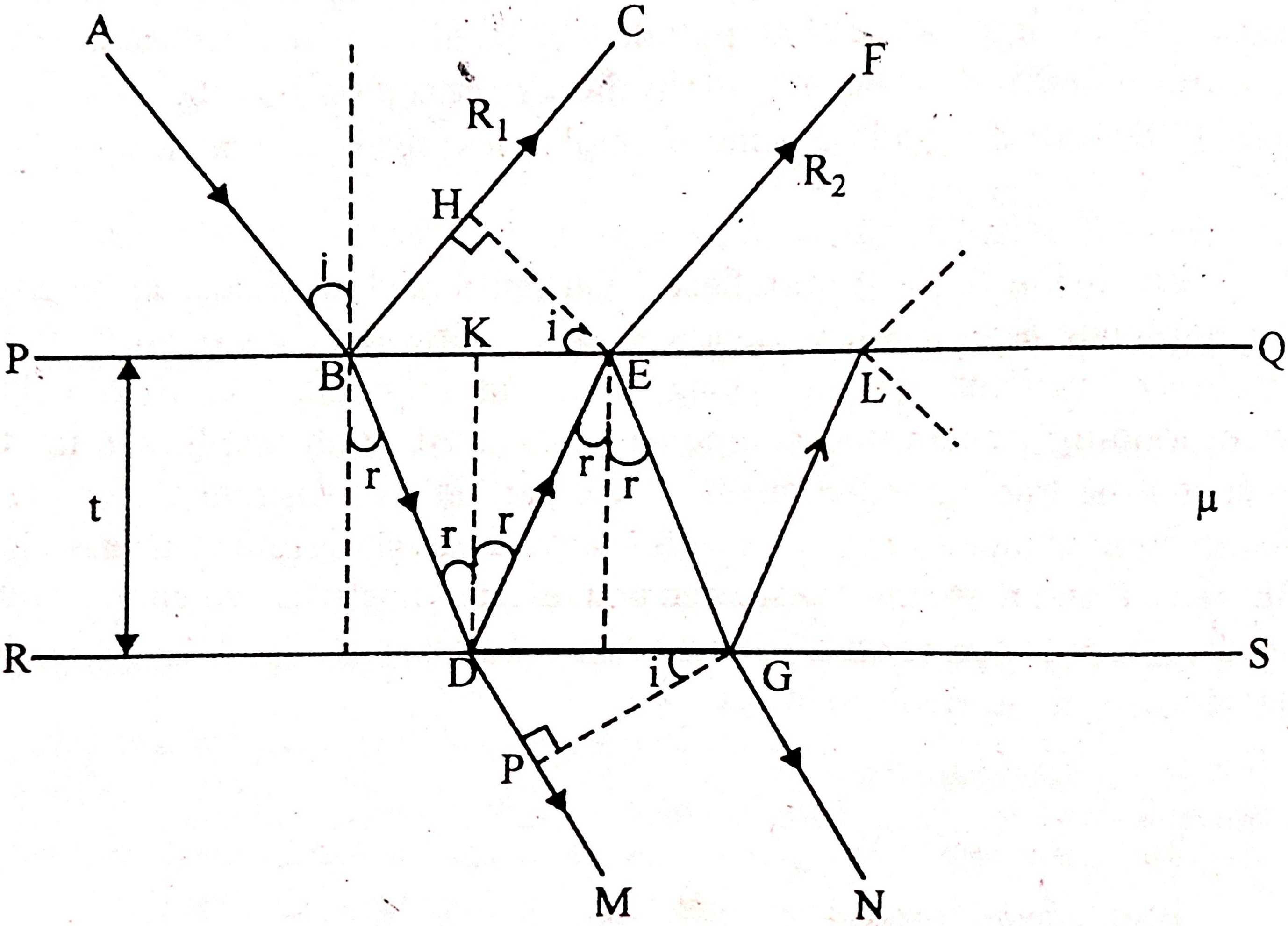

Fig. 3.13 : Interference in thin films

3.9.1 Interference in reflected light

As the rays BC and EF are derived from the same source (By the division of amplitude) therefore they are coherent and undergo interference. The reflected rays are represented by R_1 and R_2. As the film is thin and of uniform thickness, the rays R_1 and R_2 will be parallel and the interference between these two rays are known as interference in reflected light. The interference fringes so obtained will be bright or dark depending upon the path or phase difference between the interfering rays.

Now to determine the path difference between these two rays, draw a perpendicular EH on BC and DK on BE.

As the path beyond EH is the same, therefore the path difference between the two rays

$$\Delta = (BD + DE) \text{ in film} - BH \text{ in air}$$

or $$\Delta = \mu\,(BD + DE) - \mu_0\, BH$$

($\because$ Optical Path = Refractive index of the medium × actual path)

Since the refractive index of air $\mu_0 = 1$

$$\therefore \quad \Delta = \mu\,(BD + DE) - BH \qquad \text{...(38)}$$

Now in ΔBDK and in ΔEDK

$$BD = DE = \frac{DK}{\cos r} = \frac{t}{\cos r} \quad \text{...(39)}$$

and $BH = BE \sin i$

$$= (BK + KE) \sin i = 2\,BK \sin i$$

$$= 2\,DK \tan r \sin i \qquad [\because BK = DK \tan r]$$

$$= 2\,t \tan r \sin i \qquad [\because DK = t]$$

$$= 2t \frac{\sin r}{\cos r} . \sin i \qquad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$

or $$BH = \frac{2\mu t \sin^2 r}{\cos r} \quad \text{...(40)}$$

By using equations (39) and (40) in equation (38), we get

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}[1 - \sin^2 r]$$

or $\Delta = 2\mu t \cos r$ ...(41)

According to Stoke's law, if a ray is backend by a denser medium, then it undergoes an additional phase change of π (or path change of $\lambda/2$).

Hence, the effective path difference between the ray BC and EF is

$$\Delta_{eff} = 2\mu t \cos r + \frac{\lambda}{2}$$ ...(42)

Condition for maxima (bright fringe)

For maxima, the effective path difference should be equal to integral multiple of λ, i.e.

$$\Delta_{eff} = n\lambda$$ ...(43)

From equations (42) and (43), we get

or $2\mu t \cos r = (2n - 1)\lambda/2$ where $n = 1, 2, 3, \ldots$ ...(44)

Condition for minima (dark fringe)

For minima, the effective path difference should be equal to odd integral multiple of $\lambda/2$, i.e.

$$\Delta_{eff} = (2n + 1)\lambda/2 \qquad ...(45)$$

From equation (42) and (45), we get

$$2\mu t \cos r = n\lambda, \qquad \text{where } n = 1, 2, 3, \ldots \qquad ...(46)$$

3.9.2 Interference in transmitted light

As we have described in Fig. 3.13 that the two rays T_1 and T_2 i.e., DM and GN are obtained from the single ray AB by division of amplitude hence they are coherent and undergo interference on superposition and produces bright and dark fringes. The interference pattern, which we get from the superposition of T_1 and T_2 are known as interference in transmitted light. To determine the path difference between transmitted ray draw a perpendicular GP on DM.

Similarly as we have done in interference in thin film in reflected light, the path difference between the transmitted rays T_1 and T_2 is

$$\Delta = (DE + EG) \text{ in film} - DP \text{ in air}$$

$$= \mu(DE + EG) - DP$$

or $$\Delta = 2\mu t \cos r \qquad ...(47)$$

In this case there is no phase change due to reflection at D and E as the light is reflected from rarer medium. Hence the effective path difference between these rays is $2\mu t \cos r$

(i) Conditions for maxima (bright fringe)

For maxima $\Delta = n\lambda$ ...(48)

Therefore from equations (47) and (48), we get

$$2\mu t \cos r = n\lambda, \qquad \text{where } n = 0, 1, 2, 3, \ldots \qquad ...(49)$$

When this condition is satisfied, the film will appear bright in the transmitted light.

(ii) Conditions for minima (dark fringe)

For minima $\Delta = (2n + 1)\lambda/2$ ...(50)

From equations (48) and (50), we get

$$2\mu t \cos r = (2n + 1)\lambda/2, \qquad \text{where } n = 0, 1, 2, 3, \ldots \qquad ...(51)$$

When this condition is satisfied, the film will appear dark in the transmitted light.

Therefore, the conditions for maxima and minima in the reflected light are just the reverse of those in the transmitted light. Hence the film which appears bright in reflected light will appear dark in transmitted light and vice-versa.

Hence the interference pattern in reflected and transmitted light are complementary to each other.

Excessively thin film appears dark in reflected light and bright in transmitted light

As the effective path difference between the interfering waves in reflected light is $2\mu t \cos r + \lambda/2$, when the film is excessively thin such than $t << \lambda$ then $2\mu t \cos r \approx 0$

Hence effective path difference becomes $\lambda/2$. This satisfies the condition for minima (destructive interference). Hence every wavelength will be absent and the film will appear black in reflected light, while it will appear bright in transmitted light.

3.9.3 Colours in thin films

If a thin film is illuminated by monochromatic light, it will appear bright or dark depending upon the condition, which satisfy there. If the condition of maxima, i.e. $2\mu t \cos r = (2n + 1) \lambda/2$ is satisfied then the film will applear bright and if the condition of minima i.e., $2\mu t \cos r = n\lambda$ is satisfied there, then the film will applear dark.

When white light is incident on the thin film, it will appear coloured, when condition of maxima for various wavelengths is satisfied. The path differencc at any point of the film depends upon thickness of film (t) and angle of refraction (r). Since the white light consists of continuous wavelengths of seven colours and for a particular value of t and r, only certain wavelengths satisfy the condition of maxima. Therefore, only these colours will be present in the reflected light and the film will appear coloured.

Further the condition of maxima or minima depend upon t and r. Therefore if either t or r is varied, a set of different colours will be observed.

Example 8.29. *A parallel beam of light* ($\lambda = 5890 \times 10^{-8}$ cm) *is incident on a thin glass plate* ($\mu = 1.5$) *such that the angle of refraction into the plate is* 60°. *Calculate the smallest thickness of the glass plate which will appear dark by reflection.* (*Punjab 1973*)

Here $$2\,\mu t \cos r = n\lambda$$

$$\mu = 1.5, \quad r = 60^\circ, \quad \cos 60^\circ = 0.5$$

$$n = 1, \quad \lambda = 5890 \times 10^{-8}\ \text{cm}$$

$$\therefore \quad t = \frac{n\lambda}{2\,\mu \cos r} = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$t = \mathbf{3.926 \times 10^{-5}\ cm}$$

Example 8.31. *A soap film of refractive index* $\frac{4}{3}$ *and of thickness* 1.5×10^{-4} cm *is illuminated by white light incident at an angle of* 60°. *The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of* 5×10^{-5} cm. *Calculate the order of interference of the dark band.* [*Delhi* (*Hons*)]

Here,
$$2\mu t \cos r = n\lambda$$

$$\mu = \frac{4}{3} \quad ; \quad \lambda = 5 \times \times 10^{-5} \text{ cm}$$

$$t = 1.5 \times 10^{-4} \text{ cm} \quad ; \quad i = 60^\circ$$

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \frac{4}{3} = \frac{\sin 60}{\sin r}$$

$$\sin r = \frac{0.866}{4/3} = 0.6495$$

$$r = 40.5^\circ \quad \text{and} \quad \cos r = 0.7604$$

$$\therefore \qquad n = \frac{2\mu t \cos r}{\lambda}$$

$$n = \frac{2 \times 4/3 \times 1.5 \times 10^{-4} \times 0.7604}{5 \times 10^{-5}}$$

$$n = 6.0832$$

Hence, the order, **n = 6.**